CCFU Proof 24

$$\mathrm{sig}(b_{\Omega_W}) = (3, 4)$$

Given. Ω_W from Proof 22.

The Hitchin bilinear form [Proof 21, Step 4]:

$$b_{\Omega_W} = \begin{pmatrix} -6 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 3 \\ 0 & 3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 & 0 & 0 \end{pmatrix}$$

The matrix is block-diagonal: a 1×1 block (-6) and three 2×2 blocks $\left(\begin{smallmatrix} 0 & 3 \\ 3 & 0 \end{smallmatrix}\right)$, each with eigenvalues ± 3.

Eigenvalues: -6 ($\times 1$), -3 ($\times 3$), $+3$ ($\times 3$).

$$\mathrm{sig}(b_{\Omega_W}) = (3, 4). \blacksquare$$

[Dependency: Proof 22. Algebraic—hand computation.]